

Relations and Functions

- ✓ **Relation** : If A and B are two non-empty sets, then any subset R of $A \times B$ is called Relation from set A to B . i.e. $R: A \rightarrow B \Leftrightarrow R \subseteq A \times B$
 If $(x, y) \in R$ then we write xRy (read as x is R related to y) and
 If $(x, y) \notin R$ then we write $x \not R y$ (read as x is not R related to y)
- ✓ **Domain and Range of a Relation** : If R is any relation from Set A to Set B then,
 - Domain of R is the set of all first coordinates of elements of R and is denoted by $\text{Dom}(R)$.
 - Range of R is the set of all second coordinates of R and it is denoted by $\text{Range}(R)$
 A relation R on set A means, the relation from A to A i.e., $R \subseteq A \times A$
- ✓ **Empty Relation** : A Relation R in a set A is called empty relation, if no element of A is related to any element of A , i.e. $R = \emptyset \subseteq A \times A$
- ✓ **Universal Relation** : A Relation R in a set A is called universal relation each of A is related to every element of A , i.e. $R = A \times A$
- ✓ **Identity Relation** : $R = \{(x, y) : x \in A, y \in A, x = y\}$ OR $R = \{(x, x) : x \in A\}$
 A Relation R in a set A is called -
- ✓ **Reflexive Relation** : If $(a, a) \in R$, for every $a \in A$
- ✓ **Symmetric Relation** : If $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$ for all $a_1, a_2 \in A$
- ✓ **Transitive Relation** : If $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies $(a_1, a_3) \in R$ for all $a_1, a_2, a_3 \in A$
- ✓ **Equivalence Relation** : If R is reflexive, symmetric and transitive
- ✓ **Antisymmetric Relation** : A relation R in a set A is antisymmetric.
 if $(a, b) \in R, (b, a) \in R \Rightarrow a = b \forall a, b \in R$ OR aRb and $bRa \Rightarrow a = b, \forall a, b \in R$.
- ✓ **Inverse Relation** : If A and B are two non-empty sets and R be a relation from A to B , such that $R = \{(a, b) : a \in A, b \in B\}$, then the inverse of R , denoted by R^{-1} , is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$
- ✓ **Equivalence class** : Let R be an equivalence relation on a non-empty set A . For all $a \in A$, the equivalence class of ' a ' is defined as the set of all such elements of A which are related to ' a ' under R . It is denoted by $[a]$.
 i.e. $[a] = \text{equivalence class of 'a'} = \{x \in A : (x, a) \in R\}$
- ✓ **Function** : Let X and Y be two non-empty sets. Then a rule f which associates to each element $x \in X$, a unique element, denoted by $f(x)$ of Y , is called a function from X to Y and written as $f: X \rightarrow Y$ where, $f(x)$ is called image of x and x is called the pre-image of $f(x)$ and set Y is called the co-domain of f and $f(X) = \{f(x) : x \in X\}$ is called the range of f .
- ✓ **One-One or Injective Function** : A function $f: X \rightarrow Y$ is defined to be one-one if the images of distinct element of X under f are distinct;
 i.e. $x_1, x_2 \in X : f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ Otherwise f is called many-one.

✓ **Onto or Surjective** : A function $f: X \rightarrow Y$ is said to be onto if every element of Y is the image of some element of X under f ; i.e. for every $y \in Y$, there exists an element x in X such that $f(x) = y$

✓ **One - One and onto or Bijective** : A function $f: X \rightarrow Y$ is said to be one - one and onto, if f is both one - one and onto.

📍 **Note** : $f: X \rightarrow Y$ is onto if and only if $\text{Range of } f = Y$

✓ **Composition of function** : Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two function then the composition of f and g denoted by $g \circ f$ and defined as the function $g \circ f: A \rightarrow C$

$$g \circ f = g[f(x)], \quad \forall x \in A$$

✓ **Invertible function** : A function $f: X \rightarrow Y$ is defined to be invertible, if there exists a function $g: Y \rightarrow X$ such that $g \circ f = I_X$ and $f \circ g = I_Y$. The function g is called the inverse of f and is denoted by f^{-1} .

✓ **Binary operation** : A binary operation $*$ on a set A is a function $*$: $A \times A \rightarrow A$. we denote $*(a, b)$ by $a * b$

• A binary operation $*$ on a set A is called commutative, if $a * b = b * a$, for every $a, b \in A$.

• A binary operation $*$: $A \times A \rightarrow A$ is said to be associative if $(a * b) * c = a * (b * c)$, $\forall a, b, c \in A$

• A binary operation $*$: $A \times A \rightarrow A$, an element $e \in A$, if it exists, is called identity for the operation $*$, if $a * e = e * a$, $\forall a \in A$

• A binary operation $*$: $A \times A \rightarrow A$ with the identity element e in A , an element $a \in A$ is said to be invertible with respect to the operation $*$, if there exists an element b in A such that $a * b = e = b * a$ and b is called the inverse of a and is denoted by a^{-1} .

✓ **No. of function** : Let $f: A \rightarrow B$ be any mapping and $|A| = n$ and $|B| = m$ where, $|A|$ represent no. of elements in Set A
 $|B|$ represent no. of elements in Set B

Then ; Total no. of function from A to $B = m^n$

• **Case (i)** If $n = m$; then
Total no. of mapping = n^n
Total no. of one - one mapping = $n!$
Total no. of onto mapping = $n!$

• **Case (ii)** If $n < m$; then
Total no. of mapping = m^n
Total no. of one - one mapping = ${}^m C_n n!$
Total no. of onto mapping = 0

• **Case (iii)** If $n > m$; then
Total no. of mapping = m^n
Total no. of one - one mapping = 0
Total no. of onto mapping = $\sum_{r=0}^{m-1} (-1)^r {}^m C_r (m-r)^n$